

$B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$ AND THE DETERMINATION OF f_B

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We compute the rate of the decay $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$ by a QCD relativistic potential model, with the result: $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma) = 0.9 \cdot 10^{-6}$. We also discuss how this decay mode can be used to access f_B .

1 Introduction

The B meson leptonic constant f_B , defined by $\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B(p) \rangle = i f_B p^\mu$, plays a prime role in the physics of heavy-light quark systems. In principle, it can be obtained from the purely leptonic decays: $B^- \rightarrow \ell^- \bar{\nu}_\ell$, whose rates are proportional to f_B^2 . However, the helicity suppression, represented by a factor $(m_\ell/m_B)^2$, implies very low decay rates for $\ell = e, \mu$; using $V_{ub} = 3 \cdot 10^{-3}$ and $f_B = 200 \text{ MeV}$ one predicts: $\mathcal{B}(B^- \rightarrow e^- \bar{\nu}_e) \simeq 6.6 \cdot 10^{-12}$; $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu) \simeq 2.8 \cdot 10^{-7}$ ^a. This problem is absent in the τ channel, which however presents identification difficulties. One may ask whether f_B could be obtained from other decays. A candidate^{2,3} is $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$, since a third body in the final state prevents helicity suppression. Indeed, using a relativistic potential model we predict that its branching ratio is higher than in the purely leptonic channel⁴; this gives us the possibility to access f_B from this decay mode.

2 $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$ in a Relativistic Potential Model

The amplitude of the decay $B^-(p) \rightarrow \mu^-(p_1) \bar{\nu}_\mu(p_2) \gamma(k, \epsilon)$ can be written as $\mathcal{A}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma) = \frac{G_F}{\sqrt{2}} V_{ub} (L^\mu \cdot \Pi_\mu)$, where $L^\mu = \bar{\mu}(p_1) \gamma^\mu (1 - \gamma_5) \nu(p_2)$ is the weak leptonic current and $\Pi_\mu = \Pi_{\mu\nu} \epsilon^{*\nu}$ is the hadronic correlator:

$$\Pi_{\mu\nu} = i \int d^4x e^{iq \cdot x} \langle 0 | T[J_\mu(x) V_\nu(0)] | B(p) \rangle. \quad (1)$$

In (1) $q = p_1 + p_2$, $J_\mu = \bar{u} \gamma_\mu (1 - \gamma_5) b$ is the weak hadronic current and $V_\nu = \frac{2}{3} e \bar{u} \gamma_\nu u - \frac{1}{3} e \bar{b} \gamma_\nu b$ is the electromagnetic current containing the coupling of the photon both to the light and the heavy quark. The B meson is described by a wave function $\psi_B(\vec{k}_1)$ representing the distribution of the heavy quark momentum \vec{k}_1 inside the hadron. In the B rest frame it can be obtained by

^a Experimental bounds¹ are: $\mathcal{B}(B^- \rightarrow e^- \bar{\nu}_e) < 1.5 \cdot 10^{-5}$, $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu) < 2.1 \cdot 10^{-5}$.

solving numerically a Salpeter equation which includes relativistic effects in the kinematics⁵. Quark interaction is described by the Richardson potential, reproducing QCD phenomenology since it is linear for large distances, and therefore confining, and it behaves as $\frac{\alpha_s(r)}{r}$ for small r , according to asymptotic freedom. Spin interaction can be neglected, being $\mathcal{O}(m_b^{-1})$. In this framework, we obtain⁴:

$$\Gamma(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma) = \frac{G_F^2 |V_{ub}|^2}{3(2\pi)^3} \int_0^{m_b/2} dk^0 k^0 (m_b - 2k^0) [|\Pi_{11}|^2 + |\Pi_{12}|^2] \quad (2)$$

where Π_{11} and Π_{12} derive from (1)⁴. Moreover, the bound $|\vec{k}_1| \leq \frac{m_B^2 - m_u^2}{2m_B}$ is obtained assuming that the b quark has a running mass: $m_b^2(\vec{k}_1) = m_B^2 + m_u^2 - 2m_B(\vec{k}_1^2 + m_u^2)^{1/2}$, and imposing $m_b^2 \geq 0$, an equation stemming from the requirement of energy conservation at quark level⁶: $E_b + E_u = m_B$ ($E_{b,u} = (\vec{k}_1^2 + m_{b,u}^2)^{1/2}$). Finally, we put a cut-off Λ for small photon energies k^0 to avoid the unphysical divergences in (2) for $k^0 \rightarrow 0$, corresponding to no photon emitted in the final state^b. Using $\Lambda = 350 \text{ MeV}$, we obtain⁴:

$$\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma) = 0.9 \cdot 10^{-6} \quad (3)$$

a result that depends very slightly on Λ . The computed photon spectrum (Fig. 1) shows a peak around 1.5 GeV .

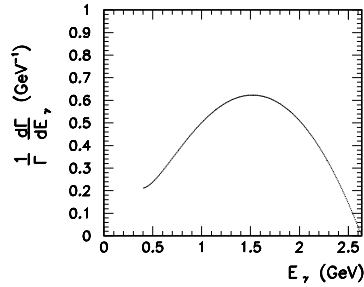


Figure 1: Photon energy spectrum in the decay $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$.

^bThe inclusion of radiative corrections would formally cancel the divergence; notice that only photon energies larger than 50 MeV are experimentally measurable.

3 How to relate f_B to $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$

The determination of f_B from $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$ relies on heavy quark symmetries. In fact, in the limit $m_b \rightarrow \infty$ f_B can be related to the B^* decay constant f_{B^*} , defined by: $\langle 0 | \bar{b} \gamma_\mu q | B^*(p, \epsilon) \rangle = f_{B^*} m_{B^*} \epsilon_\mu$, by the relation: $f_B = f_{B^*} = \frac{\hat{F}}{\sqrt{m_b}}$, with \hat{F} independent of m_b . Theoretical analyses⁷ indicate: $\hat{F} \simeq 0.35 \text{ GeV}^{3/2}$. To relate $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$ to \hat{F} , let us consider that the decay proceeds through bremsstrahlung and structure dependent diagrams. The

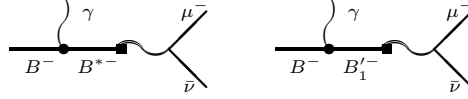


Figure 2: Polar diagrams contributing to $B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$

former vanish if one puts $m_\mu = 0$, as we do; the latter are polar diagrams (Fig. 2), the pole being either a B^* or a B_1' ; this second state is the only $J^P = 1^+$ meson which couples to the weak current in the $m_b \rightarrow \infty$ limit. We get³:

$$\begin{aligned} \Gamma(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma) &= \Gamma^{(B^*)} + \Gamma^{(B_1')} \\ &= \int_0^{\frac{m_B}{2}} \frac{2 dE_\gamma E_\gamma^3 (m_B - 2E_\gamma)}{3(2\pi)^3} \left[\frac{|C_1|^2 f_{B^*}^2}{(E_\gamma + \Delta)^2} + \frac{|C_2|^2 f_{B_1'}^2}{(E_\gamma + \Delta')^2} \right], \quad (4) \end{aligned}$$

where C_1 (C_2) depends on the coupling $g_{B^* B \gamma}$ ($g_{B_1' B \gamma}$)³. Expression (4) gives access to f_{B^*} , and hence to \hat{F} , if $\Gamma^{(B_1')} \ll \Gamma^{(B^*)}$. Using theoretical inputs to compute (4), we found: $\Gamma^{(B_1')} \simeq \Gamma^{(B^*)}/10$, concluding that the contribution of the second diagram can be included in the uncertainty on f_B . Within this approximation, $\Gamma(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ is proportional to \hat{F}^2 . We propose a method to get \hat{F} based on the experimental knowledge of both $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ and $g_{B^* B \gamma}$. The prediction $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma) \simeq \mathcal{O}(10^{-6})$ suggests that this measurement could be accessible at the future B factories^c. As for $g_{B^* B \gamma}$, it is not experimentally known, but heavy quark symmetries relate it to the analogous D coupling $g_{D^* D \gamma}$ ⁹. The measurement: $\mathcal{B}(D^{*0} \rightarrow D^0 \gamma) = 36.4 \pm 2.3 \pm 3.3\%$ is already available¹⁰. Since the bound $\Gamma(D^{*0}) < 131 \text{ KeV}$ ¹¹ is not far from current predictions⁹, it is possible that $\Gamma(D^{*0})$ will be measured in the next future. Such measurement would give $g_{D^* D \gamma}$. To show the sensitivity of the method, we used two theoretical inputs for $g_{D^* D \gamma}$. The plot of $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ versus \hat{F} is shown in Fig. 3³. The experimental result for $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ would

^cA recent analysis⁸ gives: $\mathcal{B}(B \rightarrow \mu \nu_\mu \gamma) < 9.5 \cdot 10^{-5}$; $\mathcal{B}(B \rightarrow e \nu_e \gamma) < 1.6 \cdot 10^{-4}$.

give \hat{F} . In the expected range of values of \hat{F} , the results in Fig. 3 confirm that $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ should be $\mathcal{O}(10^{-6})$.

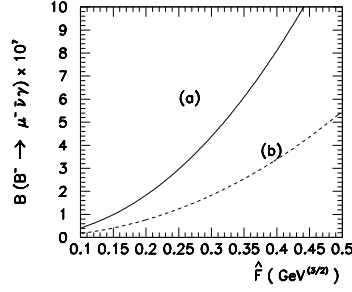


Figure 3: The branching ratio $\mathcal{B}(B^- \rightarrow \mu^- \bar{\nu}_\mu \gamma)$ versus \hat{F} ; the curves (a) and (b) refer to the values: $\Gamma(D^{*0} \rightarrow D^0 \gamma) = 22 \text{ KeV}$ and $\Gamma(D^{*0} \rightarrow D^0 \gamma) = 11 \text{ KeV}$, respectively.

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